

Cruel Teacher, Smart Student

THIS ISSUE'S PUZZLE

Dr. I.M. Meene, a high school physics teacher, is also an inventor. Most of his inventions are very cruel with respect to his students. His latest gadget was designed for lazy students who neglect doing their homework. It has three buttons—red, white, and blue—and a screen on which one can write with a stylus. The teacher pre-selects the number of times each button should be pressed and gives the device to his student. If the student presses the blue button, he needs to write the sentence “I need to do my homework diligently” on the screen as many times as he still needs to press the white button. If he presses the red button, the student must write the sentence three times the number of remaining blue button pushes. Finally, if the student presses the white button, he must write the sentence two times the number of remaining red button pushes.

Bobby has not done his latest homework and is up for punishment. Dr. Meene gave him the gadget and told him to push the blue button $A=5$ times, the red button $B=2$ times, and the white button $C=6$ times. Bobby may be lazy, but he’s certainly very astute. He figured out how to accomplish the daunting task with the lightest effort. Assuming the device can’t be tempered with and that Bobby didn’t cheat, please answer the following questions:

1. How many sentences did Bobby write?
2. In how many ways can the smallest number of sentences be written?
3. Generalization: What are the answers to questions 1 and 2 in terms of A , B , and C ? (Not required to make the solver list.)

EXAMPLE. Dr. Meene selected for a student to press the blue button four times, red button three times, and white button twice. If the victim presses the red button, he needs to write “I need to do my homework diligently” $3 \times 4 = 12$ times. Once he writes 12 sentences, the system accepts the entries and the student will need to press the red button two more times in addition to pushing the other buttons six times, in whatever order he prefers.

There is a unique solution for some larger limits on the sum $A + B$. Assume, for example, that $A + B < 65$.

The first two statements don’t provide us with any additional information.

The third one, by Green, “I don’t know the sum,” requires analysis. This expression means that A and B can’t be primes, their products, or certain powers of primes. The product of A and B must have more than two proper factors, such as $30 = 2 \times 3 \times 5$, which has many proper factors. However, some combinations are out even if there are more than two proper factors. For instance, number 363 is ruled out even though $363 = 3 \times 11 \times 11$. The only eligible combination would be 33×11 , since 121×3 won’t work as $121 + 3 = 124 > 65$.

The fourth statement by Blue, “I already knew that,” is more difficult to understand. It means that whatever the summands that add up to $A + B$ are, their products should have two or more proper factors within the conditions of the problem. In this case, if Blue had been given 21, then Green could have deduced the numbers if A and B were 7 and 14.

Unfortunately, finding such numbers that satisfy the condition in the previous sentence requires extensive mental calculations, so computer power might be useful. There are only seven such “magic” sums: 11, 17, 23, 27, 29, 35, and 37.

Green then goes on to say, “I now know the sum.” Before this statement, Green deduced that the sum is in the set shown above. Now, by saying that he knows the sum, he tells us (and Blue) that there is only one combination of factors of the product of A and B , the sum of which is “magic.” There are 45 different possibilities.

Why so few? If the product isn’t in the range of possibilities, it means that none of the summands from possible factorizations add up to 11, 17, ..., or there is more than one combination of possible factors, the sum of which belongs to the “magic” sum set.

Blue finally replies, “I now know the

PREVIOUS ISSUE PUZZLES

More Logicians

Two brilliant, perfect logicians, Green and Blue, are given the following information regarding the two positive integers, A and B : They are both greater than 1, $A < B$, and their sum is less than 40.

Green is told the product of A and B , and Blue is given the sum of A and B . The logicians are then engaged in the following conversation:

Green: “I know the product.”
 Blue: “I know the sum.”
 Green: “I don’t know the sum.”
 Blue: “I already knew that.”
 Green: “I now know the sum.”
 Blue: “I now know the product.”

Please answer the following questions:

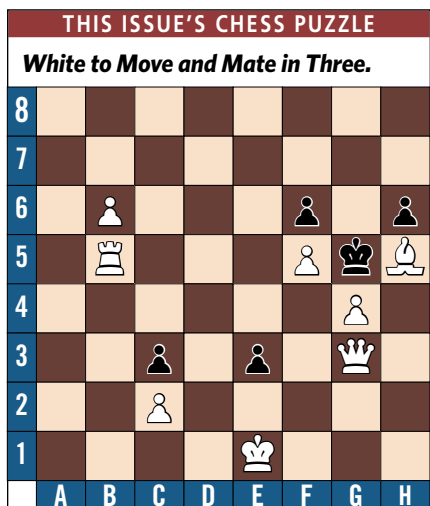
1. What are the numbers A and B ?
2. Is there only one solution?

Solution

1. 4 and 13
2. Yes

The problem is widely known, so the solution could be found on the Web or in a variety of mathematical books.

By making $A + B < 40$, I actually made the problem unsolvable. Thanks to everyone who pointed this error out to me.



product." This means that out of 45 possible combinations, there is only one with the unique sum. This works only when A + B = 17 with A = 4 and B = 13.

Chess Puzzle

White to move and mate in three.

Initial position:

White—Kf5, Nc5, Ne4, pawns c3, d6.

Black—Kd5, pawns c4, c6, c7, e5.

Solution

1. Kf6 cxd6
2. Ke7 dxc5
3. Nf6#

SOLVER LISTS

Due to an administrative deadline, names of only those people who submitted cor-

rect solutions by July 31, 2006, are shown on the lists.

Logician Puzzle: Bob Bartholomew, Richard Bottelli, Bob Byrne, Bob Campbell, Alan Clark, Mike Crooks, William Cross, Mark Danburg-Wyld, Mark Evans, Michael Failor, Rui Guo, John Hubenschmidt, Raja Malkani, Stephen Meskin, Lee Michelson, Jeff Mobley, Don Onnen, Jim Pederson, Stephen Peeples, Luke Porter, David Promislow, Mark Proska, Philip Silverman, Martin Smith, Al Spooner, Elnatan Sulimanoff, John Tomlinson, Kevin Trapp, David Uhland, Christian Ulmer, Matt Wickwire

Chess Puzzle: Steve Altschuld, Steven Azar, Robert Burrell, Greg Campbell, Hobson Carroll, Mike Crooks, Jason Douridas, Daly Dowd, Jeff Dvinoff, Leigh Halliwell, Norman Henricks, Ignace Kuchazik, Philip

Lehpamer, Brian Liebeskind, Raja Malkani, John Marshall, John McCarthy, June Meimban, Mark Mercier, Lee Michelson, Don Onnen, Harry Ploss, Dave Scherr, Noam Segal, Don Sondergeld, Tim Swankey, Kevin Trapp, Lee Zinzow

Solutions may be e-mailed to cont_puzzles@yahoo.com or mailed to **Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.**

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by **Sept. 30, 2006.** Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

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