How Long a Shot Is Powerball?

EVERYONE KNOWS THE ODDS ARE STACKED AGAINST HIM when he plays the lottery. But just how stacked? You can answer that question by determining the expected value of Powerball.

Powerball is a lottery game with large jackpots and smaller cash prizes. It features five white balls numbered 1 through 55 and one red Powerball numbered 1 through 42. The white balls and the red balls are independent of one another. In order to win the jackpot, you must match the numbers for all five white balls and the one red Powerball. Other prizes are awarded based on varying combinations of white and red balls as listed below. There is also a "Powerplay" option available to the player who doubles the cost of the ticket, from $1 to $2, but multiplies any non-jackpot prize by a randomly chosen multiplier between two and five. (As a note, this article was written in July 2008, and any changes in the structure of the game after that time will alter our analysis.)

The expected value of Powerball isn't always constant. Jackpot sizes start at $15 million and increase after each drawing until won, and the jackpot is divided equally among all winners. So even though the odds of the game remain constant, the expected value of a Powerball ticket varies depending on jackpot size and the number of people playing. We were interested in determining under what circumstances, if any, buying a Powerball ticket would have a positive expectation. We were also interested in analyzing the impact of the Powerplay option.

KYLE MROTEK is a consulting actuary with Milliman in Brookfield, Wis. TOM VAN HOUTTE is an actuarial student at Milliman.

Methodology

We made some important assumptions in this analysis that should be noted.

A. First, all jackpot values represent the stated jackpot prior to a Powerball drawing and not the actual amount received by the player. According to the Powerball website (www.powerball.com), the stated jackpot is estimated as follows. Of every dollar sold, 30 cents goes into the cash jackpot pool and accumulates until there is a winner or winners. If winners choose the cash option, they receive the accumulated cash jackpot pool. If winners choose the annuity option, the accumulated cash jackpot pool is invested and they receive 30 annuity payments, increasing by 4 percent each year over 29 years. The stated jackpot is the estimated amount of the 30 annuity payments made over 29 years.

FIGURE 1

Powerball Odds and Prizes

Result		Prize	Probability	Odds	Expected Value Excluding Jackpot
Correct Number of White Balls	Powerball	(1)	(2)	(3) = [1 /(2)] -1	(4) = (1) × (3)
5	1	Jackpot	0.000	146,107,961 : 1	
5	0	200,000	0.000	3,563,608 : 1	0.036*
4	1	10,000	0.000	584,431 : 1	0.011*
4	0	100	0.000	14,253 : 1	0.007
3	1	100	0.000	11,926 : 1	0.008
3	0	7	0.003	290 : 1	0.024
2	1	7	0.001	744 : 1	0.009
1	1	4	0.008	126 : 1	0.032
0	1	3	0.015	68 : 1	0.044
< 3	0	0	0.973	1 : 36	0.000
					0.171

*Adjusted for taxes

ISTOCKPHOTO

B. Second, if a jackpot is won, the winner takes the cash option up front. We looked at cash option proportions to stated jackpots since 2007, and found that they fell in a range from 46 percent to 50 percent, depending on interest rates at the time. For the purpose of this analysis, we estimate the cash option will be 48 percent of the stated jackpot. Using 48 percent of the stated jackpot implies a 4.9 percent annual return on the lump-sum payment to keep up with the annuity option. It's not necessarily a better choice to take the cash option, but since that is the choice that an overwhelming majority of jackpot winners take, we have assumed that for our analysis.

C. Third, prize amounts will be treated on an after-tax basis, assuming a 35 percent tax rate. Thus, the winner of a jackpot who takes the lump-sum amount will receive approximately 31 percent of the stated jackpot after taxes. We don't factor in state income taxes.

D. Finally, there's also a "Match 5 Bonus Prize" that we ignored. Whenever a Powerball jackpot reaches a record level, the jackpot is limited to a maximum of a $25 million increase. The money collected in excess of the $25 million maximum increase is placed in the Match 5 prize pool and accumulates until there's a jackpot winner. At that point, the Match 5 prize is divided equally among all players who matched five white balls but not the Powerball. The expected value of this prize is zero as long as the jackpot is not at a record level. Determining the expected

FIGURE 2 **Probability of Exact Number of Winning Tickets**

	Number of Tickets Sold				
Number of Winning Tickets	5 Million	25 Million	50 Million	75 Million	100 Million
0	0.966	0.843	0.710	0.599	0.504
1	0.033	0.144	0.243	0.307	0.345
2	0.001	0.012	0.042	0.079	0.118
3	0.000	0.001	0.005	0.013	0.027
4	0.000	0.000	0.000	0.002	0.005
5	0.000	0.000	0.000	0.000	0.001
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000
Sum	1.000	1.000	1.000	1.000	1.000

FIGURE 3 **Expected Value of Jackpot Given at Least One Winner**

		Number of Tickets Sold				
Number of Winning Tickets	Portion of Jackpot Won	5 Million	25 Million	50 Million	75 Million	100 Million
1	100%	0.983	0.917	0.839	0.765	0.697
2	50%	0.008	0.039	0.072	0.098	0.119
3	33%	0.000	0.001	0.005	0.011	0.018
4	25%	0.000	0.000	0.000	0.001	0.002
5	20%	0.000	0.000	0.000	0.000	0.000
6	17%	0.000	0.000	0.000	0.000	0.000
7	14%	0.000	0.000	0.000	0.000	0.000
8	13%	0.000	0.000	0.000	0.000	0.000
Total Expectation of Jackpot		0.991	0.958	0.916	0.876	0.836

value when the jackpot is at a record level would require obtaining sales information we don't have access to or predicting sales behavior. The effect of this assumption has been largely insignificant to the purpose of this paper.

Analysis

To get the overall expected value, first we calculated the non-jackpot expectation, as this is always constant. Figure 1 shows that, excluding the possibility of winning the jackpot, one would expect to get back 17.1 cents for every dollar spent on Powerball.

In order to determine the expected value of a given jackpot size, we first needed to determine the number of winners. This was done through basic combinatorial calculations. Since the number of winners will vary depending on how many people are actually playing, Figure 2 shows the results for several sales scenarios.

Using this distribution, we were able to obtain the expected portion of the jackpot one winner would receive, given that the jackpot has been won, based on varying ticket-sales scenarios. For example, if 25 million tickets were sold, a jackpot winner can expect to receive only about 95.8 percent of the stated jackpot, because of the possibility that it would be split with other winners. Figure 3 shows this distribution.

We got the jackpot expectation by multiplying the expected portion of the jackpot won for varying ticket sales and jackpot sizes by the odds of winning the jackpot. By combining the non-jackpot and jackpot expectations, we came up with a total expectation for a Powerball ticket based on ticket sales and jackpot size. For example, Figure 4 shows that a player would expect a return of $1.04 for a $1 Powerball ticket with a stated jackpot of $450 million and with 50 million tickets sold. The $1.04 consists of:

(Non-jackpot expectation) + [(jackpot size) × (cash option, after tax value) × (probability of winning jackpot) × (expected portion of jackpot won)]

= $.171 + ($450,000,000 × .31 × 1/146,107,962 × .916) = $1.04

The same calculations were done assuming the Powerplay option was chosen. The only change for Powerplay is the non-jackpot expectation, as jackpot amounts aren't multiplied. Since the Powerplay multiplier is a random number between two and five, we just multiplied the non-jackpot expectation by 3.5 and doubled the cost of the ticket. As Figure 5 indicates, this significantly decreases expectation.

Conclusions

While it's theoretically possible, Powerball will rarely, if ever, have a positive expectation. As of July 2008, the largest Powerball jackpot ever was $365 million on Feb. 16, 2006. Yet it would take jackpots in excess of $400 million to potentially reach a positive expectation. Considering the extremely high variance associated with Powerball, even if it was a positive expectation, a better bet could probably be found. For instance, most casino games expect to pay out over 90 percent of the money that is wagered. Powerball will rarely reach that level of return.

High ticket sales decrease expectation but probably not very significantly. For instance, holding the jackpot constant at $150 million, the difference in expectation

FIGURE 4 Expected Value of One Powerball Ticket—No Powerplay

Number of Winning Tickets (Millions)	Jackpot Size ($ Millions)				
	25	150	300	450	600
5	(0.78)	(0.51)	(0.20)	0.12	0.43
25	(0.78)	(0.52)	(0.22)	0.08	0.39
50	(0.78)	(0.54)	(0.25)	0.04	0.34
75	(0.78)	(0.55)	(0.27)	0.01	0.28
100	(0.78)	(0.56)	(0.30)	(0.03)	0.23

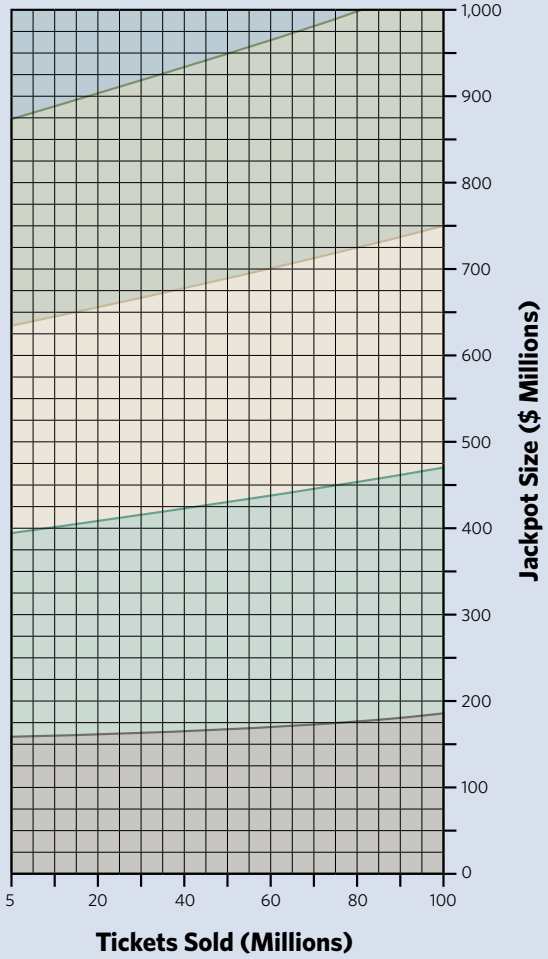
FIGURE 5 Expected Value of One Powerball Ticket—With Powerplay

Number of Winning Tickets (Millions)	Jackpot Size ($ Millions)				
	25	150	300	450	600
5	(1.35)	(1.09)	(0.77)	(0.46)	(0.14)
25	(1.35)	(1.10)	(0.79)	(0.49)	(0.18)
50	(1.35)	(1.11)	(0.82)	(0.53)	(0.24)
75	(1.35)	(1.12)	(0.84)	(0.57)	(0.29)
100	(1.36)	(1.13)	(0.87)	(0.60)	(0.34)

ISTOCKPHOTO

FIGURE 6

Expected Value of One Powerball Ticket—No Powerplay



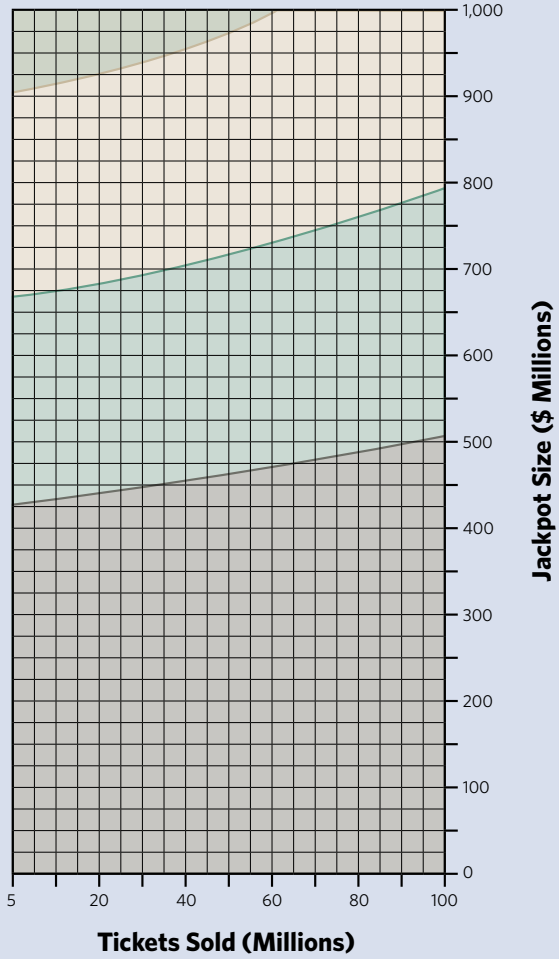
Expected gain/loss on a $1 ticket
Break-even point is $0.00

-$1.00 to -$0.50 | $0.00 to $0.50 | $1.00 to $1.50

-$0.50 to $0.00 | $0.50 to $1.00

FIGURE 7

Expected Value of One Powerball Ticket—With Powerplay



Expected gain/loss on a $2 ticket
Break-even point is $0.00

-$1.00 to -$0.50 | $0.00 to $0.50

-$0.50 to $0.00 | $0.50 to $1.00

between 5 million and 100 million tickets sold is only about four cents.

Powerplay isn't a fair bet. It would take jackpots larger than $700 million to approach a positive expectation with Powerplay. It's probably safe to assume that there will never be a positive expectation with the Powerplay option as it is currently defined.

Figures 6 and 7 illustrate the expectation in more detail. The shaded regions represent ranges of expected gains/losses of a Powerball ticket based on varied ticket sales and jackpot-size scenarios. For instance, the region at the bottom of the graph includes all points at which a player would expect to lose at least 50 cents for each Powerball ticket played, whereas the top region includes all points at which a player would expect a profit of at least $1. The line between $400 million and $500 million in Figure 6 represents the points at which a player would expect to break even on the purchase of a Powerball ticket. The positive slope of each region represents the decreased expectation as ticket sales increase. ●