Remarkable Parabola

THIS ISSUE'S PUZZLE

A mathematician, tired after a day full of productive research at his university, decided to relax for a while doing… some more math work. He drew a parabola with 0 and 1 as x-intercepts and a maximum of 1 at x = ½. He then picked a number between 0 and 1 on the x-axis and looked to see what the corresponding y-coordinate was. That y-coordinate became his new x-coordinate, and the mathematician looked at the new y-coordinate. The process continued, and to his surprise, the fourth x-coordinate (which was non-zero) did not change from the third. Obviously, the process went into an infinite loop after that.

What was his starting x-coordinate? (Note that a closed-form solution is needed.)

The mathematician was so excited that he soon developed a generic formula for the starting position if x-coordinates start to repeat after n moves. Can you do the same?

PREVIOUS ISSUE PUZZLES

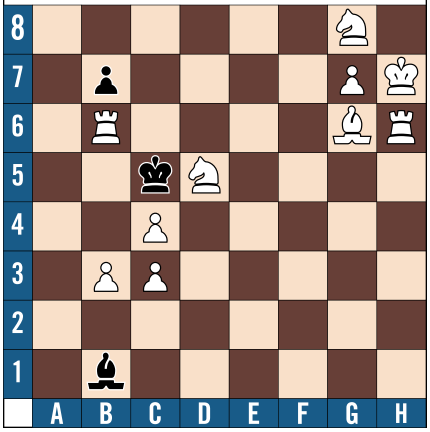
A Game in the School Corridor

A narrow school corridor needs to be filled with 100 square tiles, all placed in one line. The tiles are identical in shape and could be either black or white. Tile placeholders are clearly marked on the floor.

The job to fill the corridor with tiles of alternating color is given to two students in detention. They decide to play the following game instead: Each player in turn puts either a black or a white tile into an empty square. The game stops when three adjacent squares contain a black, a white, and a black tile in that order and the last player wins. If all the squares are filled without getting a black, a white, and a black tile in that order, then the game is a draw.

THIS ISSUE'S CHESS PUZZLE

White to Move and Mate in Three.



Assuming that there is an unlimited supply of tiles of each color, which player can always win?

Solution

The second person can always win.

Suppose a square marked on the floor is such that if one of the students puts a tile there, that allows his opponent to win on his next move. If a student puts a white tile in, then his opponent must win by placing an adjacent black tile. So we must have B-1-2-3, where B stands for a black tile, 1 and 2 are empty, and a student puts a white tile on square 1. But he also loses if he places a black tile, so his opponent must then win by putting a white tile on 2, which means that 3 must already contain a black tile. The situation is symmetrical, so that 2 is also a losing square. Thus, until someone plays on one of them, losing squares always occur in pairs.

The board has an even number of squares, so the first player always faces the floor with an even number of squares not yet occupied, whereas the second player always faces the floor with an odd number of squares not yet occupied. Therefore, provided (1) there is at least one pair of losing squares, (2) he never plays on a losing square, and (3) he makes the obvious winning move if the first player ever creates the opportunity, then the second player is sure to win, because the first player will eventually face the floor with only losing squares available to put a tile in.

To make sure there is at least one pair of losing squares, the second player must create it. He can always do this by placing a black tile on his first move well away from the first player's move and from the edges of the floor. Then on his second move (assuming the first player has not allowed him an immediate win), he can always put another black tile three away from it, creating a pair of losing squares. Thereafter, he must simply take care to win if there is a winning move and otherwise to avoid losing plays.

Last Issue's Chess Puzzle

White to move and mate in three.

Initial position: White – Kf8, Qa8, pawns c4, d3, e3, g4. Black – Kf6, Nf1, pawns a7, c5, e6, f7, g5.

Solution—Case A

1. Qh1 Nh2
2. Qa1+ e5
3. Qa6#

Solution—Case B1

1. Qh1 Ke5
2. Ke7 f5
3. Qh8#

Solutions—Case B2

1. Qh1 Ke5
2. Ke7 Ng3
3. Qa1#

Solutions—Case B3

1. Qh1 Ke5
2. Ke7 Kg6
3. Qh8#

Solutions—Case C

1. Qh1 all other black moves
2. Qh8+ Kg6
3. Qg7#

SOLVER LISTS

Due to an administrative deadline, names of only those people who submitted correct solutions by Jan. 31, 2007, are shown on the lists.

Corridor Puzzle: *Bob Bartholomew, Bob Byrne, Bill Carroll, Mike Failor, Rui Guo, Yehuda Haber, John Hubenschmidt, Sharon Kuester, Lee Michelson, David Oakden, Stephen Peeples, Yan Peng, Steve Powell, David Promislow, Phil Rohrer, Levi Self, Leonid Shteyman, Al Spooner, Tony Torelli, Kevin Trapp*

Chess Puzzle: *Robert Burrell, Bill Carroll, Alan Erlebacher, Leigh Halliwell, Krishna Kothoor, Philip Lehpamer, June Meimban, Lee Michelson, Harry Ploss, Jason Russ, Levi Self, Kevin Trapp, Lee Zinzow*

Solutions may be e-mailed to cont_puzzles @yahoo.com or mailed to Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by March 31, 2007. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

ADVERTISER INDEX

To add your company's name to this list, call Mohanna & Associates at 800-800-0341 or e-mail info@mohanna.com.

For links to these advertisers' e-mail addresses and websites, visit the *Contingencies* website at www.contingencies.org/linksto_advert.html.