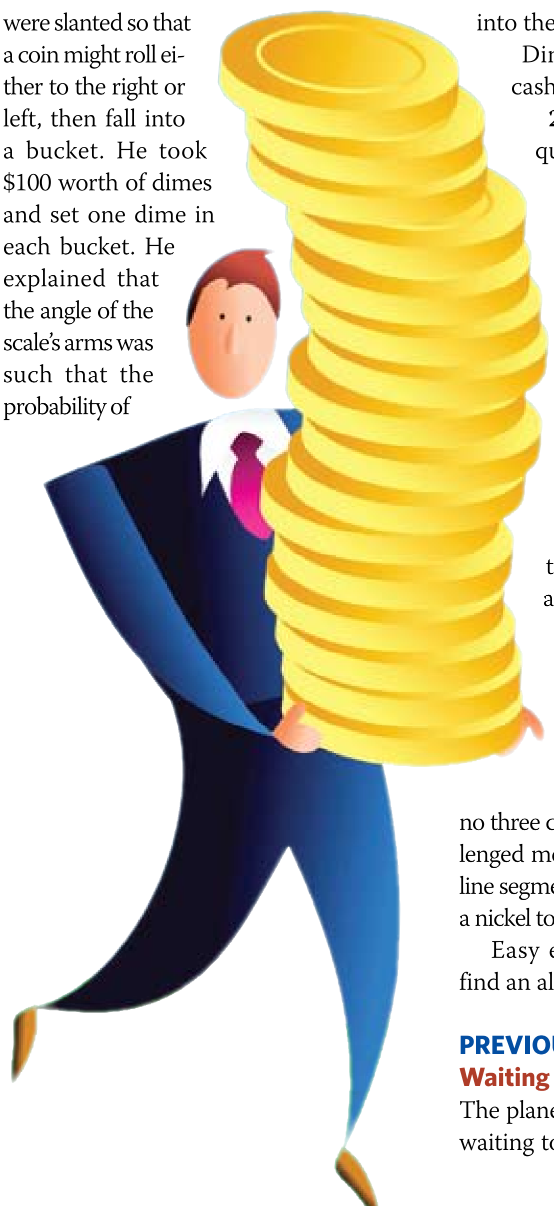
Coin Tricks

THIS ISSUE'S PUZZLE

I spent the weekend visiting an old friend, Mr. Maxwell Chance. He's an eccentric sort, mostly harmless, but he had recently developed a numismatic obsession and so playing with coins took up most of our time. This lead to the following three "quickie" puzzles:

1. The Scales: Max set up a coin slot that led down to a pair of balance scales. The arms of the scales were slanted so that a coin might roll either to the right or left, then fall into a bucket. He took $100 worth of dimes and set one dime in each bucket. He explained that the angle of the scale's arms was such that the probability of a coin rolling right or left was proportional to the number of coins already in each bucket. For example, if the right bucket had 13 coins and the left bucket had four, the probability of the next coin rolling to the right was 13/17.



DAWN HUDSON / 123RF.COM

To start off, Max placed a single dime in each bucket (to prevent the first roll from completely deciding the remaining outcome).

"What would you give me now in advance," he asked, "for the right to the lighter bucket after I roll the remaining 998 dimes into the scales?"

Dimes spend as well as any other cash, so what is a fair bid?

2. The Chessboard: Max set out quarters down the diagonal of a chessboard, then played a (somewhat tedious) game of laying out additional coins based on the rule that if an empty square shared two or more sides with a filled square, it should also be filled. The board quickly filled in.

"The diagonal is kind of a special case," he explained. "If I lay them out at random, I usually need nine or 10 to be sure of filling in the board. I think eight is the minimum, but I haven't been able to prove it."

Can you?

3. The Table: Max took $2.82 in nickels and pennies, and threw them onto a table that he had covered with a large sheet of paper. After carefully checking that no three coins fell onto a single line, he challenged me to connect the coins by straight line segments, such that each line connected a nickel to a penny and no two lines crossed.

Easy enough—but your puzzle is to find an algorithm that always works.

PREVIOUS ISSUE PUZZLE

Waiting Game

The plane had been sitting on the tarmac, waiting to take off, for almost 30 minutes when the guy in the seat next to me challenged me to a game of tick-tack-toe. Mind you, we had not spoken a word to each other up to that point, so I was a bit startled; but I couldn't think of a reason to decline, so I agreed. We played three games in the next couple of minutes, and I won all three.

"Well, I can see you are pretty good at this game," my companion said. "What do you say we make it a little more interesting and put some money on this next one?"

"What do you have in mind?" I asked.

"I'll bet you a dollar that you don't beat me in our next game."

I may not be exactly streetwise, but I can recognize a hustle when I see one—even if, as in this case, the hustler is only 8 years old. I paused, as if considering.

"Tell you what," I said after a minute, "to keep this fair, let's make it a game of chance. We'll number the cells 1 through 9, and also write the numbers 1 through 9 on some slips of paper. Then we can draw numbers at random to determine where we get to go on our turn. You can go first. If you win, I pay you a dollar; otherwise, you pay me a dollar."

He thought this over and agreed.

So the question is, was that a fair bet? Or is my wife correct that I was ripping off an 8-year-old? What is the probability of the first player winning in a randomized game of tick-tack-toe? And, while you are at it, what is the probability of a tie game?

Solution

There are 9!/(5!4!) = 126 possible outcomes which fall into four cases:

1. 16 ties
2. 62 wins for X
3. 12 wins for O
4. 36 that might be a win for either player, and thus require further analysis.

In case 4, for X to win after three moves has odds of 1/10. O wins on his third move 9/40 of the time (if X didn't win on the previous move, O wins 25 percent of the time, so 9/10 * 1/4 = 9/40). X wins on his fourth move 9/40 of the time (X completes the first tick-tack-toe on his fourth move 30 percent of the time but wins only if O didn't complete tick-tack-toe on the previous move, so 3/10 * 3/4 = 9/40). O wins on his fourth move 9/20 of the time (if X hasn't completed a tick-tack-toe yet, which happens 60 percent of the time, and O didn't complete a tick-tack-toe earlier, which happens 75 percent of the time, so 3/5 * 3/4 = 9/20). X can't win on his fifth move, because we know O has a tick-tack-toe that must have been competed no later than O's fourth move. So, X wins 1/10 + 9/40 =13/40 of the games.

Therefore, X wins with probability 62/126 + (13/40)*(36/126) = 737/1260 (~58.5%).

A draw happens with probability 16/126 (~12.7%).

O wins with probability 12/126 + (27/40)*(36/126) = 121/420 (~28.8%).

A number of solvers used brute-force search algorithms and expressed their solutions as fractions of 362,880. ●

Solutions may be e-mailed to cont.puzzles@gmail.com or mailed to Puzzles, 65 W. 35th Place, Eugene, Ore. 97405.

In order to make the solver list, please make sure that your answers and solutions are received by July 31, 2008. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

SOLVER LIST:

Solvers—*Jon Anderson, Steven Azar, Robert Bartholomew, Geoff Bridges, Bob Byrne, Raymond Chiang, Bill Cross, Lance Dyrland, Ryan Elmore, David Engelmayer, Yan Fridman, Yehuda Haber, John Hubenschmidt, Alex Kozmin, Matt Lynch, Lee Michelson, Geoff Moak, Philip Morse, Paul Navratil, Chris Norman, David Promislow, Damon Raben, Craig Schmid, John Snyder, Al Spooner, David Thaller, Kevin Trapp*